Maths for Computing
Tutorial 8

1. Give a combinatorial proof of the following equations, i.e., prove that both sides are counting the same thing.

a) $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.$

b) $\sum_{j=0}^{k} \binom{n}{j} = \sum_{j=0}^{k} \binom{n-1-j}{k-j} 2^j.$

c) $\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}.$

2. Prove that $\frac{(2n)!}{n!^2}$ is even if n is a positive integer. Further prove that $\frac{(2n)!}{n!^2} < 5^n$.

3. A grade school class has three sports teams. For any two students in the class, there is at least one team so that the two students are members of that team. Prove that there is a team that contains at least 2/3 of the students of the class.

4. Let $p = p_1p_2\ldots p_n$ be a permutation of $[n]$ and assume that $n \geq 3$. We say that i is an excedance of p if $p_i > i$. Compute the number of permutations of $[n]$ whose excedance set contains at least one of $n-2$ and $n-1$.

5. Give a combinatorial proof of

$$n! = \binom{n}{0}D(n) + \binom{n}{1}D(n-1) + \binom{n}{2}D(n-2) + \ldots + \binom{n}{n}D(0).$$